

Introduction to the Robustness of GeoGebra

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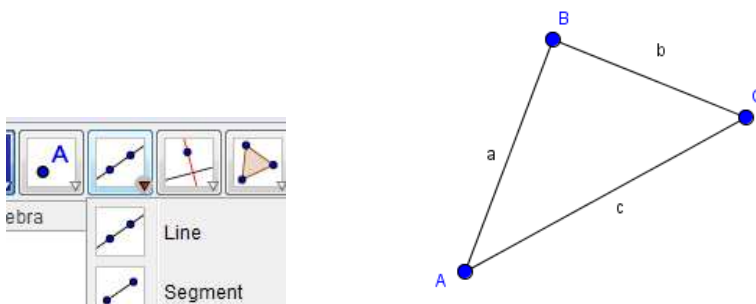
Geometry

While there are command line options, GeoGebra has many intuitive icons for making dynamic Geometry constructions.

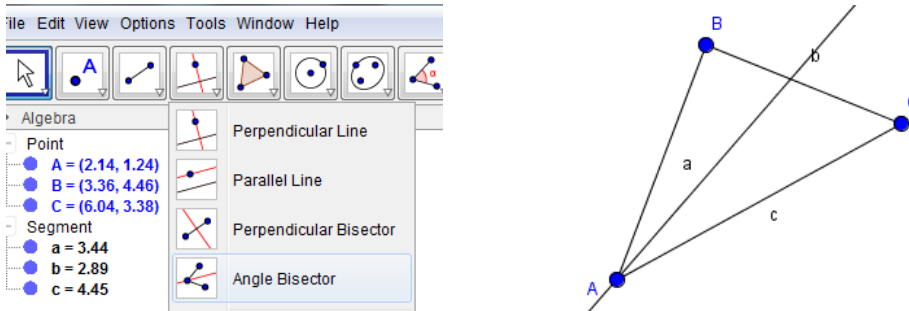
Task 1 - Construct the 4 “classic” triangle centers: Centroid (intersection of medians), Circumcenter (intersection of perpendicular bisectors), Orthocenter (intersection of altitudes), Incenter (intersection of angle bisectors)

Steps to Construct Incenter and Incircle

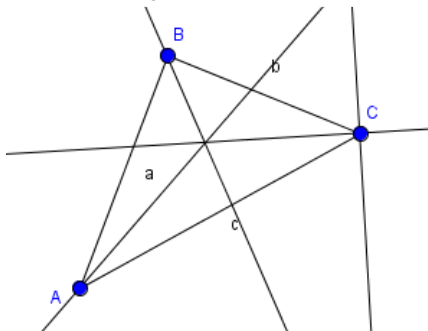
1. Using the “Segment” tool, create a dynamic triangle ABC .



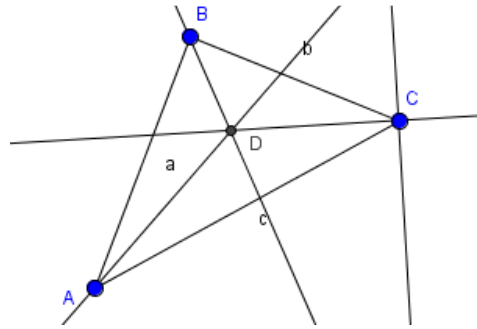
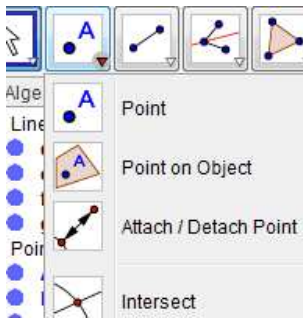
2. Select the “Angle Bisector” tool and click on points B , A , and then C to bisect angle BAC .



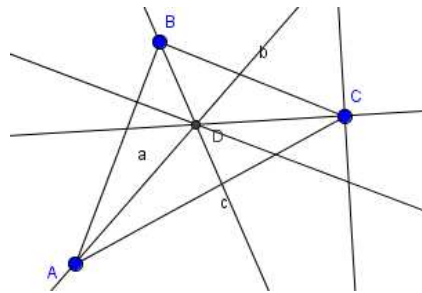
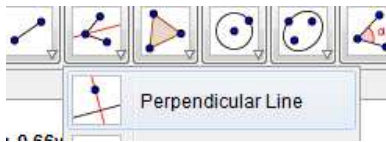
3. Use the “Angle Bisector” tool two more times to create the bisectors of angles ABC and ACB .



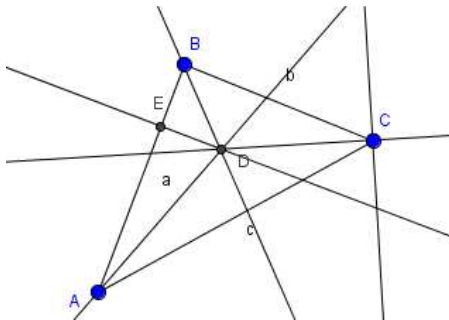
4. To create the incenter, select the “Intersect” tool, and click on any two of the angle bisectors.



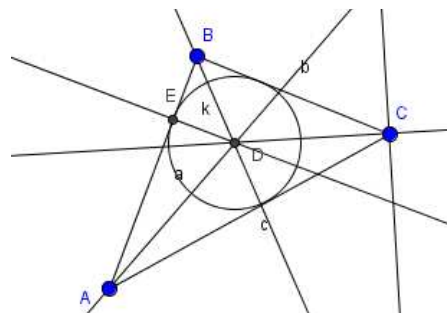
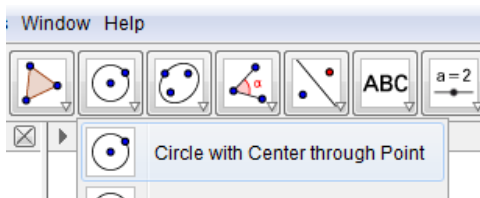
- To find the incircle, we need a point on one of the sides of the triangle that is closest to the incenter. Select the “Perpendicular Line” tool; click on the incenter; click on side AB.



- Select the “Intersection” tool; click on the line from step 5; click on side AB.



- Select the “Circle with Center through Point” tool; click on the incenter; click on the intersection point from step 6.

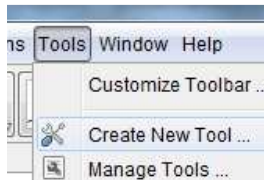


- Use the “Move” tool to drag the vertices and see the incenter and incircle update dynamically.

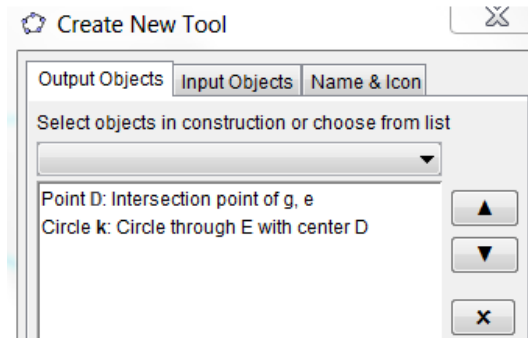


Steps to Create and Incircle & Incenter Tool

1. Using the construction file created previously, go to the “Tools” tab and select “Create New Tool”.

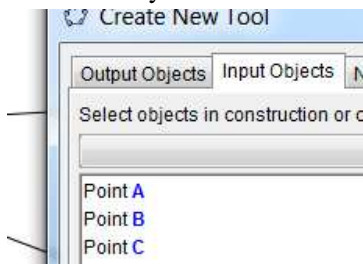


2. Select the incenter and incircle as the desired output.

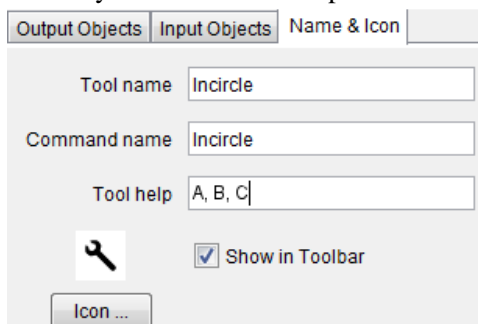


You may select the objects from the drop down menu or click on them in graphic view.

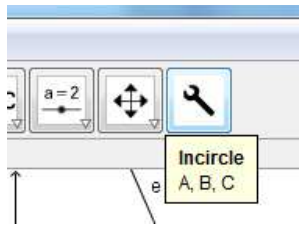
3. Click next. The appropriate input (i.e., the three vertices of the triangle) should be automatically selected.



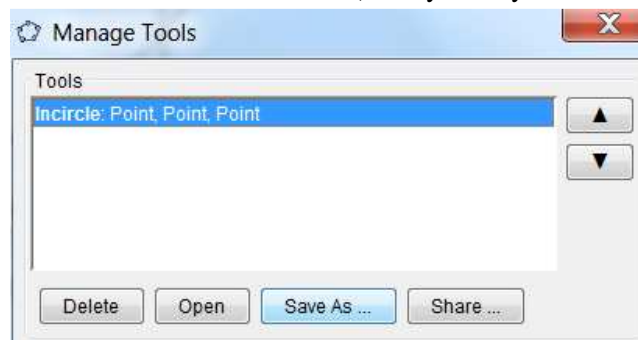
4. Click next. Name the tool with an informative name such as “Incircle”. You can add notes to remind you of the needed input in the “Tool help” box.



5. Click “Finish”, and the tool should appear after the other icons on the tool bar.



6. To save the tool,
go to “Manage Tools” in the “Tools” tab,
make sure the tool is selected (note: you may create multiple tools in a single file),



click “Save As” and save the file as “incircle.ggt”.

Notes: GeoGebra construction files have the ending “.ggb” while tools have the ending “.ggt”. You can open a tool by opening the ggt file in any other GeoGebra file.

Task 2: Create the Gergonne Point for a triangle (<http://mathworld.wolfram.com/GergonnePoint.html>)

Task 3: Create the Symmedian Point for a triangle (<http://mathworld.wolfram.com/SymmedianPoint.html>)

Task 4: Create a dynamic quadrilateral $ABCD$. Find the midpoints M , N , O , and P of sides AB , BC , CD , and AD respectively. Construct quadrilateral $MNOP$. What type of figure is this?

Algebra

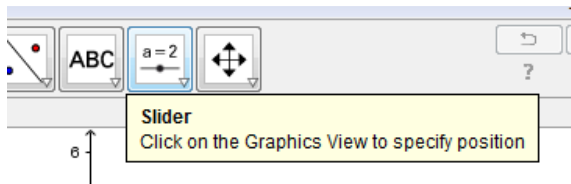
Sliders allow users to define a variable with values that can be changed by dragging a slider tool. These are very useful for exploring the effects of parameters on families of functions.

Task 1: Create a file to explore $y = a(x - h)^2 + k$

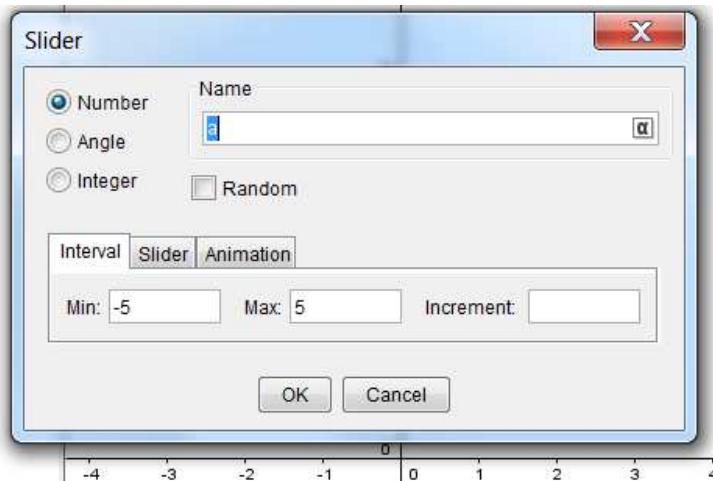
1. Make a new GGB file with three sliders a , h , and k . Let each run from -5 to 5 by 0.1.

Creating a slider:

- Select the slider tool from the toolbar.

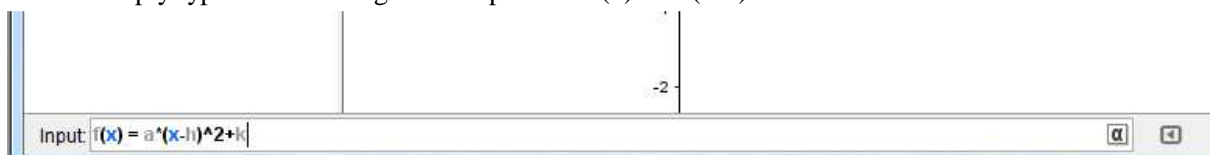


- Click where you want to place the slider in the graphics view, and a dialog box will open.



- Type the desired name; set the minimum, maximum, and increment; and click "OK".
2. Plot the function $f(x) = a(x-h)^2 + k$.

Simply type the following in the input bar: $f(x) = a*(x-h)^2+k$



3. Adjust the value of a to determine the effect of a on the graph. What happens when a changes signs? What happens as $|a|$ increases?
4. Adjust the values of h to determine the effect of h on the graph. Describe this.
5. Adjust the values of k to determine the effect of k on the graph. Describe this.

Task 2: Create a file to explore $y = ax^2 + bx + c$

1. Make a GGB file with sliders a , b , and c that all go from -10 to 10 by 0.1. Then plot the function $f(x) = a*x^2 + b*x + c$.
2. Vary the value of a with the slider. What effect does a have on the graph?
3. Vary the value of c with the slider. What effect does c have on the graph?
4. What effect does b have on the graph? Add the minimum to the graph (use Extremum[f]). Then right click on the minimum and select “trace on.” Now vary the value of b . What shape is traced out by the vertex? Find a formula for the function that is traced out. Add that function to the graph. Once the function is added remove the traces of the vertex.
5. Make a check box to turn the function added in part d off and on.

Task 3: Fitting a polynomial to intercepts

Find an equation for a polynomial with x -intercepts $x = -6$, -1 , and 4 and y -intercept at $y = 8$. Make a sketch with this function. Plot the x -intercepts as red points, the y -intercept as a blue point, and all extrema as green points.

Task 4: Maximizing an area with fixed perimeter

1. Make a sketch with a slider that adjusts to show all given rectangles of perimeter 9 units along with the area. Note that the area is automatically added when you create a polygon.
2. Use the sketch to find the dimensions that give the maximum area.
3. Use calculus to show that the rectangle with maximum area given a perimeter of 9 units is a square.

Trigonometry

Task 1: The Sine Graph

- Plot the unit circle by typing the input: $x^2 + y^2 = 1$
- Make a slider named a that runs from 0 to 4π with $\pi/12$ increments. We want radians instead of degrees so make it a “number” slider, not an “angle.”
- Plot the point $(\cos(a), \sin(a))$. You should be able to rotate the point around the circle with the slider.
- Plot the point $(a, \sin(a))$.
- Make a segment connecting the two points.
- Turn the trace on for point B . Move the slider around the circle and see the sine graph mapped out.

Task 2: The Right Triangle Trigonometric Ratios

- Plot the point $A=(0,0)$
- Plot a point B somewhere in quadrant I
- Plot the point $C=(x(B),0)$
- Use the polygon tool to construct triangle A, B, C .
- Place a point D on the positive x-axis (not on C).
- Construct angle DAB . Then hide point D . You should now be able to move B to any quadrant and see a triangle with one side on the x-axis, a vertex at zero, and an angle between 0° and 360° .
- Type each of the following in the text boxes to get some measurements:
 $\text{opp} = y(B)$ $\text{adj} = x(B)$ $\text{hyp} = \text{Distance}[A,B]$
- Now add the following text to the graphic view by typing each in the input bar:
“opposite =”+opp “adjacent =”+adj “hypotenuse =”+hyp
- Add text to show the computation of $\sin(a)$ by typing:
“ $\sin(a) = \text{opposite/hypotenuse} =$ ”+opp+”/”+hyp+”=”+(opp/hyp)
- Add similar text to show the computations of $\cos(a)$ and $\tan(a)$.

Task 3: Modeling solutions to a trigonometric equation

Solve the trigonometric equation $(2\sin(x)-1)(2\cos(x)-1) = 0$. Graph $f(x) = (2\sin(x)-1)(2\cos(x)-1)$. Make a slider k running from -10 to 10 by increments of 1. Then add the solutions as points controlled by the slider.

Task 4: Finding a Temperature Function

The following table gives the monthly average high temperatures in Atlanta.

Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
52	57	65	73	80	87	89	88	82	73	63	55

1. Create a new Geogebra file.
2. Open the “spreadsheet view” under the view tab. Adjust the window sizes as necessary to see the algebra, graph, and spreadsheet views at the same time.
3. Enter 1 through 12 in the column *A* for the 12 months.
4. Enter the corresponding average high temperatures in column *B*.
5. Highlight the data (left click, hold, drag, release), then right click and select “Create List of Point.” You should see the list in the algebra view. The points are also plotted, but you need to adjust the window.
6. Right click on an axis (make sure the points are deselected) and select “Graphics view.” Adjust the min and max of the x-axis and y-axis so that all of the points are visible.
7. Create sliders a (min = -20, max = 20), b (min = -5, max = 5), k (min = -5, max = 5), and d (min = -100, max = 100).
8. Plot the function $y = a*\cos(k*(x-b)) + d$.
9. Adjust the slider values until the function fits the points as nicely as possible (it won't fit exactly).
10. Record your best fit function with the numerical values of the sliders plugged into it.
11. Use your model to predict the average high temperature in August. How close is the estimate to the actual value? (You can evaluate the function with Geogebra.)

Calculus

Task 1: Plotting secant and tangent lines

Create a file with sliders for a , b , and c . Then add the following:

1. $f(x) = a*x^2 + b*x + c$
2. a point P on the parabola
3. a slider h running from -1 to 1 by .01
4. a point $Q = (x(P)+h, f(x(P)+h))$ (This is a point “close to P ”.)
5. the secant line between P and Q .
6. the tangent line at P

You should see that the secant line approaches the tangent as $h \rightarrow 0$.

Task 2: Left Hand Sums

Construct the left hand sums and integral.

1. Enter the function: $f(x) = x^2 + 1$
2. Create sliders a (running from 0 to 5 by .1), b (running from a to 5 by 0.1), and n (running from 1 to 50 by 1)
3. Enter: $dx = (b-a)/n$
4. Enter: $L = \text{Sequence}[\text{Polygon}[(i, f(i)), (i+dx, f(i)), (i+dx, 0), (i, 0)], i, a, b-dx, dx]$
5. Enter: $\text{LHS} = \text{Sum}[\text{Sequence}[dx*f(a+i*dx), i, 0, n-1]]$
6. Enter: $\text{integral}[f, a, b]$

Statistics

Descriptive Statistics: Box Plots, Dots Plots, and Histograms

Task 1: Create the list of data

1. Open the file data.ggb. It contains a set of (made up) test scores.
2. Select all of the data by clicking on the column label.
3. Right click, go to “create,” and create a list.
4. Right click on list1 in the algebra view and rename it L.

Task 2: Making a dot plot

1. In the input bar type: $D = \text{DotPlot}[L]$
This will create a new list of points. They are plotted as a dot plot, but you can't see them unless you adjust the axes. Since they are exam scores, none are below zero and none are above 100.
2. Right click somewhere on the axes and select “graphics” at the bottom of the options.
3. Set the x Min to -10 and the x Max to 110. You should then be able to see the dot plot. A dot plot simply plots a point above any score that is made. If the same score occurs more than once, you see a stack of points at that x value.
4. Is the distribution roughly symmetric, skewed left, or skewed right?

Task 3: Finding some numerical values

1. To find the “average” (sum of the scores divided by the number of scores), type the following in the input bar: $\text{mean}L = \text{mean}[L]$
2. To find the sample standard deviation, type: $\text{sd}L = \text{sampleSD}[L]$
3. The mean and the standard deviation are good descriptors of a data set when the distribution is roughly symmetric and bell shaped. If not, the five number summary (min, q1, median, q3, max) is more informative.
4. Find the minimum test score by typing: $\text{min}L = \text{min}[L]$
5. Find the maximum test score by typing: $\text{max}L = \text{max}[L]$
6. The first or lower quartile, Q1, is the value at which 25% of the scores lie at or below. To find this type: $\text{q1}L = \text{Q1}[L]$
7. The median is the value at which 50% of the scores lie above and 50% lie below. To find this type: $\text{med}L = \text{median}[L]$
8. The third or upper quartile, Q3, is the value at which 75% of the scores lie at or below. To find this type: $\text{q3}L = \text{Q3}[L]$
9. Is the five number summary or the mean and standard deviation more appropriate for this data?

Task 4: Making a box plot

Geogebra has several different types of possible inputs to make a box plot. We will look at two of them. The first will use the actual raw data and the second will use the 5 number summary.

One way to construct a box plot is: $\text{BoxPlot}[\langle y\text{Offset} \rangle, \langle y\text{Scale} \rangle, \langle \text{list of raw data} \rangle]$

The yOffset argument controls where the center of the boxplot will be in the y-direction. The yScale argument controls how thick the box and whiskers will be. It will be the distance from the center of the boxplot to the bottom or top of the box.

1. Change the y-axis so that it will be tall enough to place two box plots above the dot plot.
2. Make a box plot above the dot plot using the raw data.

Another way is to use the 5 number summary instead of the data:
BoxPlot[<yOffset>, <yScale>, min, Q1, median, Q3, max]

3. Make another box plot above the first one using the 5 number summary. Note the two box plots will be the same (except for the yOffset). You are just showing that you could make one either way.

Task 5: Making a frequency table and histogram

1. Open a clean copy of data.ggb.
2. Use the data to create a list, and name it L as before.
3. Create a list of boundary values: $B = \{0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$
4. To create a frequency table type: `FrequencyTable[B, L]`
The result is a table that tells how many of the data points fall in each interval. Note that each interval is of the form $[a, b)$ except the last one which is $[a, b]$. So for example, a 60 is counted in the interval $60 - 70$.
5. Move the table to the lower right portion of the graph screen. Right click on the table and select "Absolute Position on Screen." This will keep the table in place when you change the scale.
6. Before we make a histogram, we need to adjust the scale. As with the dot and box plots, set the x Min to -10 and the xMax to 110.
7. To make a histogram, type: `Histogram[B, L, false]`
The last command tells it to use the frequencies instead of densities.
8. Adjust the graph as necessary to see the entire histogram and the frequency table.

Task 6: You are going to use Geogebra to see if there is any relationship between smoking and education level. 1. The file smoking.ggb contains information from 37 different public health units in Canada. Each row represents data from a different location. Column A is the percent of people surveyed by that health care unit that were smokers. Column B is the percent of the people who reported having some post secondary education. Column C is the percent of the people who reported having graduated high school.

1. Create a list of points using column B as the x-coordinate and column A as the y-coordinate.
 $L1 = \text{Sequence}[(\text{Cell}[2,i], \text{Cell}[1,i]), i, 2, 38]$
2. Does the percent of people who smoke tend to increase or decrease as the percent of people with post secondary education increases?
3. Add the best fit line to the graph. There is a tool under the line tools, or you can use `fitline[L1]`.
4. What is the slope of this line? Give a real world interpretation of what this number tells us in this setting.
5. Use the line to predict the percent of smokers in a region where 50 percent of the adults have some post secondary education.
6. Hide the scatterplot and line for L1. (Click on the blue circles.)
7. Create a new list L2 that uses column C for x and column A for y.
8. Find the best fit line for L2.
9. How does the relationship between secondary education and smoking compare to the relationship between post secondary education and smoking?

Please note that GeoGebra also has tools for inferential statistics (e.g., t-tests and confidence intervals). We will not have time to look at those today.